IIT-JAM (PH)

Assignment-01

Oscillations, Waves and Optics

Simple Harmonic Motion
Damped and Forced Vibrations

QUANTA

1. (NAT) A simple pendulum hanging vertically in an elevator has a time period T_0 when the elevator is stationary. If the elevator moves upward with an acceleration of $a = 0.2$ g, the time period of oscillations is T_1. Here g is the acceleration due to gravity. The ratio $\frac{T_0}{T_1}$ is_____. (rounded off to two decimal places). [IIT-JAM 2023]

2. A particle is subjected to two simple harmonic motions along the x and y axes, described by $x(t) = a\sin(2\omega t + \pi)$ and $y(t) = 2a\sin(\omega t)$. The resultant motion is given by [IIT-JAM 2022]

   (a) $\frac{x^2}{a^2} + \frac{y^2}{4a^2} = 1$ (b) $x^2 + y^2 = 1$

   (c) $y^2 = x^2\left(1 - \frac{x^2}{4a^2}\right)$ (d) $x^2 = y^2\left(1 - \frac{y^2}{4a^2}\right)$

3. Consider the following differential equation that describes the oscillations of a physical system:

$$\alpha\frac{d^2y}{dt^2} + \beta\frac{dy}{dt} + \gamma y = 0$$

   If α and β are held fixed, and γ is increased, then, [IIT-JAM 2021]
   (A) the frequency of oscillations increases
   (B) the oscillations decay faster
   (C) the frequency of oscillations decreases
   (D) the oscillations decay slower

4. (MSQ) An object executes simple harmonic motion along the x direction with angular frequency ω and amplitude. The speed of the object is 4 cm/s and 2 cm/s when it is at distances 2 cm and 6 cm respectively from the equilibrium position. Which of the following is/are correct? [IIT-JAM 2020]

   (a) $\omega = \sqrt{\frac{3}{8}}$ rad/s (b) $\omega = \sqrt{\frac{5}{6}}$ rad/s

   (c) $a = \sqrt{\frac{140}{3}}$ cm (d) $a = \sqrt{\frac{175}{6}}$ cm

5. (MSQ) A thin rod of uniform density and length $2\sqrt{3}$ m is undergoing small oscillations about a pivot point. The time period of oscillation (T_m) is minimum when the distance of the pivot point from the center-of-mass of the rod is x_m. Which of the following is/are correct? [IIT-JAM 2020]

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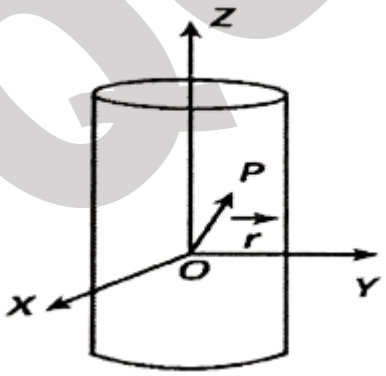
(a) $x_m = 1$ m
(b) $x_m = \frac{\sqrt{3}}{2}m$
(c) $T_m = \frac{2\pi}{\sqrt{3}}S$
(d) $T_m = \frac{2\pi}{\sqrt{5}}$ s

6. (MSQ) Three sinusoidal waves of the same frequency travel with the same speed along the positive x-direction. The amplitudes of the waves are $a, \frac{a}{2,}$ and $\frac{a}{3}$ and the phase constants of the waves are $\frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$, respectively. If A_m and φ_m are the amplitude and phase constant of the wave resulting from the superposition of the three waves, which of the following is \are correct? [IIT-JAM 2020]
(a) $A_m = \frac{5}{6}a$
(b) $\varphi_m = \frac{\pi}{2} + \tan^{-1}\left(\frac{3}{4}\right)$
(c) $A_m = \frac{7}{6}a$
(d) $\varphi_m = \tan^{-1}\left(\frac{2}{3}\right)$

7. For an underdamped harmonic oscillator with velocity $v(t)$, [IIT-JAM 2019]
(A) Rate of energy dissipation varies linearly with $v(t)$
(B) Rate of energy dissipation varies as square of $v(t)$
(C) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$
(D) For weak damping, the amplitude decays exponentially to zero

8. A particle P of mass m is constrained to move on the surface of the cylinder under a force $-k\vec{r}$ as shown in the figure (k is the positive constant). Which of the following statements is correct? (Neglect friction.) [IIT-JAM 2018]

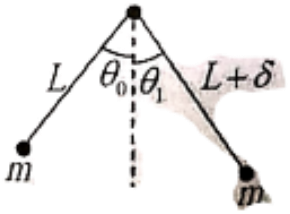


(a) The total energy of the particle is not conserved.
(b) The motion along z direction is simple harmonic.

(c) Angular momentum of the particle about O increases with time.
(d) Linear momentum of the particle is conserved.

9. Consider two waves $y_1 = a\cos(\omega t - kz)$ and $y_2 = a\cos[(\omega + \Delta\omega)t - (k + \Delta k)z]$. The group velocity if the superposed wave will be ($\Delta\omega \ll \omega$ and $\Delta k \ll k$) [IIT-JAM 2018]
(a) $\frac{(\omega-\Delta\omega)}{(k-\Delta k)}$ (b) $\frac{(2\omega-\Delta\omega)}{(2k-\Delta k)}$
(c) $\frac{\Delta\omega}{\Delta k}$ (d) $\frac{(\omega+\Delta\omega)}{(k+\Delta k)}$

10. A pendulum is made of a massless string of length L and a small bob of negligible size and mass m. It is released making an angle $\theta_0 (\ll 1\text{rad})$ from the vertical. When passing through the vertical, the string slips a bit from the pivot so that its length increases by a small amount $\delta (\delta \ll L)$ in negligible time. If it swings up to angle θ_1 on the other side before starting to swing back, then to a good approximation which of the following expressions is correct? [IIT-JAM 2017]



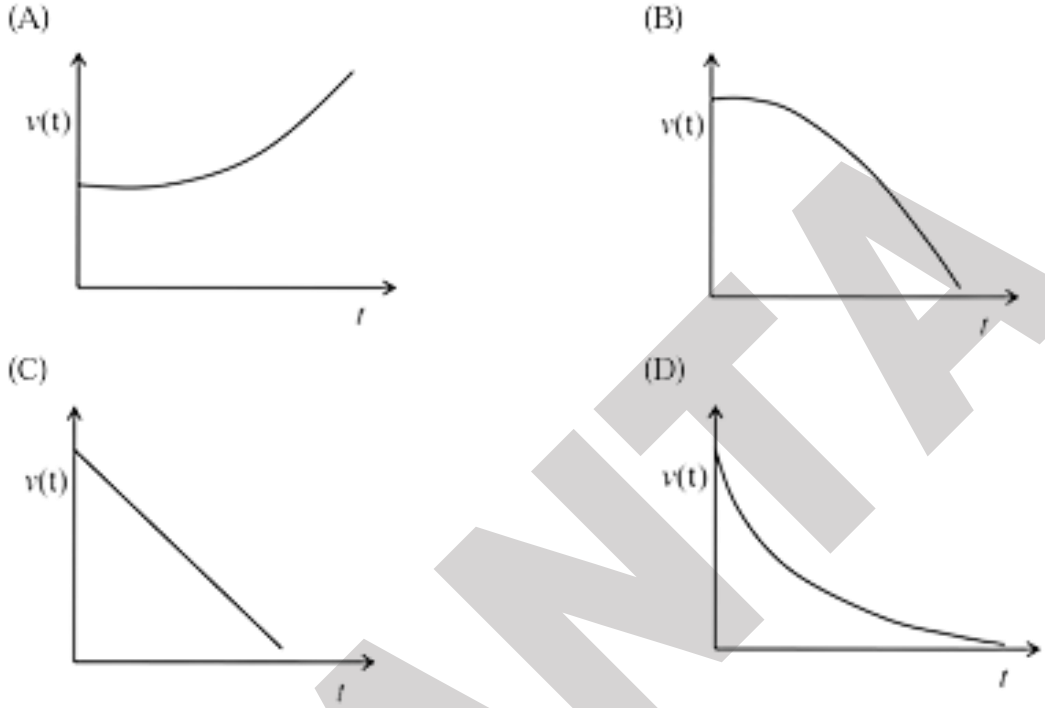
(a) $\theta_1 = \theta_0$ (b) $\theta_1 = \theta_0\left(1 - \frac{\delta}{2L}\right)$
(c) $\theta_1 = \theta_0\left(1 - \frac{\delta}{L}\right)$ (d) $\theta_1 = \theta_0\left(1 - \frac{3\delta}{2L}\right)$

11. (NAT) A particle of unit mass is moving in a one-dimensional potential $(x) = x^2 - x^4$. The minimum mechanical energy (in the same units as $V(x)$) above which the motion of the particle cannot be bounded for any given initial condition is_____. (Specify your answer to two digits after the decimal point) [IIT-JAM 2017]

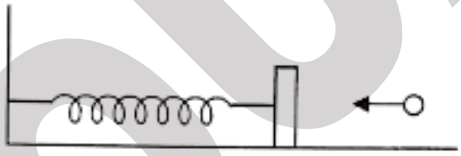
12. A lightly damped harmonic oscillator with natural frequency ω_0 is driven by a periodic force of frequency ω. The amplitude of oscillation is maximum when [IIT-JAM 2016]
(A) ω is slightly lower than ω_0.
(B) $\omega = \omega_0$.
(C) ω is slightly higher than ω_0.
(D) the force is in phase with the displacement.

13. A particle travels in a medium along a horizontal linear path. The initial velocity of the particle is v_o and the viscous force acting on it is proportional to its instantaneous

velocity. In the absence of any other forces, which one of the following figures correctly represents the velocity of the particle as a function of time? [IIT-JAM 2016]



14. A block of mass 0.38 kg is kept at rest on a frictionless surface and attached to a wall with a spring of negligible mass. A bullet weighing 0.02 kg moving with a speed of 200 m/s hits the block at time $t = 0$ and gets stuck to it. The displacement of the block (in meters) with respect to the equilibrium position is given by [IIT-JAM 2016] (Spring constant = 640 N/m)

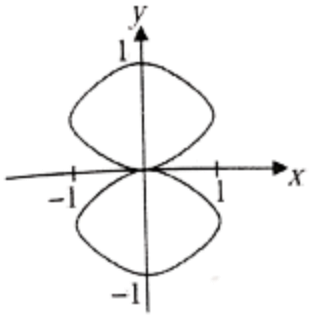


(a) $2\sin 5t$ (b) $\cos 10t$
(c) $0.4\cos 25t$ (d) $0.25\sin 40t$

15. Consider a particle of mass m following a trajectory given by $x = x_0 \cos \omega_1 t$ and $y = y_0 \sin \omega_2 t$, where x_0, y_0, ω_1 and ω_2 are constants of appropriate dimensions. The force on the particle is [IIT-JAM 2016]
(a) central only if $\omega_1 = \omega_2$
(b) central only if $x_0 = y_0$ and $\omega_1 = \omega_2$

(c) always central
(d) central only if $x_0 = y_0$ and $\omega_1 \neq \omega_2$

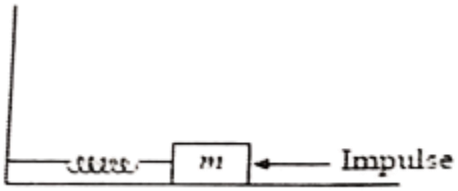
16. Two sinusoidal signals of frequency ω_x and ω_y having same amplitude are applied to x - and y - channels of a cathode ray oscilloscope (CRO), respectively. The following stationary figure will be observed when. [IIT-JAM 2016]



(a) $\omega_y = \omega_x$
(b) Phase difference is 0
(c) $\omega_y = 2\omega_x$
(d) Phase differences is $\pi/2$

17. Vibrations of diatomic molecules can be represented as those of harmonic oscillators. Two halogen molecules X_2 and Y_2 have fundamental vibrational frequencies $v_x = 16.7 \times 10^{12}$ and $v_y = 26.8 \times 10^{12}$ Hz, respectively. The respective force constants are $K_x = 325$ N/m and $K_y = 446$ N/m. The atomic masses of F, Cl and Br are 19.0,35.5 and 79.9 atomic mass unit respectively. The halogen molecules X_2 and Y_2 are [IIT-JAM 2015]
(a) $X_2 = F_2$ and $Y_2 = Cl_2$
(b) $X_2 = Cl_2$ and $Y_2 = F_2$
(c) $X_2 = Br_2$ and $Y_2 = F_2$
(d) $X_2 = F_2$ and $Y_2 = Br_2$

18. A mass m, lying on a horizontal, frictionless surface, is connected to one end of a spring. The other end of the spring is connected to a wall, as shown in the figure. At t = 0 , the mass is given an impulse [IIT-JAM 2015]



The time dependence of the displacement and the velocity of the mass (in terms of

nonzero constants A and
B) are given by
(a) $x(t) = A\sin\omega t, v(t) = B\cos\omega t$
(b) $x(t) = A\sin\omega t, v(t) = B\sin\omega t$
(c) $x(t) = A\cos\omega t, v(t) = B\sin\omega t$
(d) $x(t) = A\cos\omega t, v(t) = B\cos\omega t$

19. A traveling pulse is given by $f(x,t) = A\exp\left(\frac{2abxt - a^2x^2 - b^2t^2}{c^2}\right)$ where A, a, b and c are positive constants of appropriate dimensions. The speed of the pulse is [IIT-JAM 2013]
(a) $\frac{b}{a}$ (b) $\frac{2b}{a}$
(c) $\frac{cb}{a}$ (d) $\frac{b}{2a}$

20. A particle is released at x = 1 in a force field $\vec{F}(x) = \left(\frac{2}{x^2} - \frac{x^2}{2}\right)\hat{e}_x, x \geq 0$. Which one of the following statements is FALSE? [IIT-JAM 2013]
(a) $\vec{F}(x)$ is conservative
(b) The angular momentum of the particle about the origin is constant
(c) The particle moves towards $x = \sqrt{2}$
(d) The particle moves towards the origin

21. A traveling pulse is given by $f(x,t) = A\exp\left(\frac{2abxt - a^2x^2 - b^2t^2}{c^2}\right)$ where A, a, b, and c are positive constants of appropriate dimensions. The speed of the pulse is [IIT-JAM 2013]
(a) $\frac{b}{a}$ (b) $\frac{2b}{a}$
(c) $\frac{cb}{a}$ (d) $\frac{D}{2a}$

22. A lightly damped harmonic oscillator loses energy at the rate of 1% per minute. The decrease in amplitude of the oscillator per minute will be closest to [IIT-JAM 2012]
(a) 0.5% (b) 1% (c) 1.5% (d) 2%

23. Six simple harmonic oscillations each of same frequency and equal amplitude are superposed. The phase difference between any two consecutive oscillations i.e., $\phi_n - \phi_{n-1} = \Delta\phi$ is constant, where ϕ_n is the phase of the nth oscillation. If the resultant amplitude of the superposition is zero, what is the phase difference $\Delta\phi$? [IIT-JAM 2011]
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) 2π

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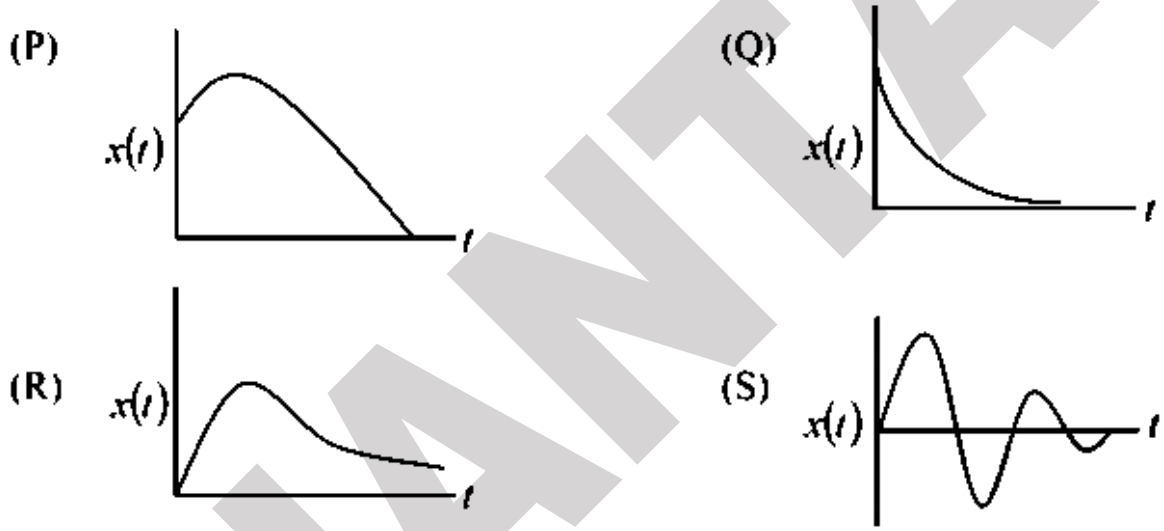
24. A particle of mass m is moving in a potential $v(x) = \frac{1}{2}mw_0^2x^2 + \frac{a}{2mx^2}$ where ω_0 and a are positive constants. The angular frequency of small oscillations for the simple harmonic motion of the particle about a stable minimum of the potential $V(x)$ is [IIT-JAM 2010]

(a) $\sqrt{2}\omega_0$ (b) $2\omega_0$
(c) $4\omega_0$ (d) $4\sqrt{2}\omega_0$

25. Among the following displacement versus time plots, which ones may represent an over-damped oscillator? [IIT-JAM 2009]



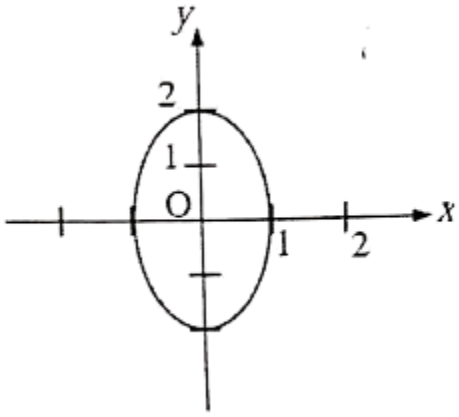
(a) only (P) and (Q) (b) only (P) and (R)
(c) only (P) and (S) (d) only (P), (R) and (S)

26. The instantaneous position $x(t)$ of a small block performing one-dimensional damped oscillations $x(t) = Ae^{-rt}\cos(\omega t + a)$. Here ω is the angular frequency, γ the damping coefficient, A the initial amplitude and α the initial phase. If $x|_{t=0} = 0$ and $\left.\frac{dx}{dt}\right|_{t=0} = v$, the values of A and α (with $n = 0,1,2$) are [IIT-JAM 2008]

(a) $A = \frac{v}{2\omega}, \alpha = \frac{(2n+1)\pi}{2}$
(b) $A = \frac{v}{co}, \alpha = n\pi$
(c) $A = \frac{v}{\omega}, \alpha = \frac{(2n+1)\pi}{2}$
(d) $A = \frac{2v}{\omega}, \alpha = \frac{(2n+1)\pi}{2}$

27. When two simple harmonic oscillations represented by $X = A_0\cos(\omega t + \alpha)$ and $y = B_0\cos(\omega t + \beta)$ are superposed at right angles, the resultant is an ellipse with its major

axis along the y-axis as shown in the figure. The conditions which correspond to this are [IIT-JAM 2007]



(a) $\beta = \alpha + \frac{\pi}{2}; A_0 = 2B_0$
(b) $\beta = \alpha - \frac{\pi}{4}; A_0 = B_0$
(c) $\beta = \alpha + \frac{\pi}{2}; 2A_0 = B_0$
(d) $\beta = \alpha + \frac{\pi}{4}; A_0 = B_0$

28. The trajectory of a particle that undergoes simple harmonic motion on a plane is shown in the figure. The ratio of the frequencies for the motion along x and y directions is given by [JEST 2022]

A. $\frac{3}{5}$ B. $\frac{2}{3}$
C. $\frac{3}{2}$ D. $\frac{4}{5}$

29. A large box, of volume V is fitted with a vertical glass tube of cross-sectional area A, in which a metal ball of mass m fits exactly. The box contain an ideal gas at a pressure slightly higher than atmospheric pressure P because of the weight of the ball. If the ball is displaced slightly from equilibrium, find the angular frequency ω of simple harmonic oscillations. Assume adiabatic behavior, with the ratio of specific heats $\gamma = C_P/C_V$. [JEST 2021]

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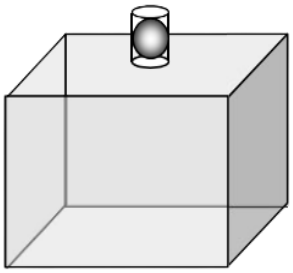
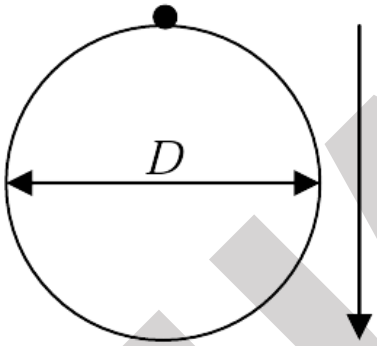


(a) $\omega = \sqrt{\frac{A^2(P+mg/A)}{2\gamma Vm}}$ (b) $\omega = \sqrt{\frac{2\gamma A^2(P+mg/A)}{Vm}}$

(c) $\omega = \sqrt{\frac{A^2(P+mg/A)}{\gamma Vm}}$ (d) $\omega = \sqrt{\frac{\gamma A^2(P+mg/A)}{Vm}}$

30. A hoop of diameter D is pivoted at the topmost point on the circumference as shown in the figure. The acceleration due to gravity g is acting downwards. What is the time period of small oscillations in the plane of the hoop? [JEST 2019]



(a) $2\pi\sqrt{\frac{D}{2g}}$ (b) $2\pi\sqrt{\frac{5D}{6g}}$

(c) $2\pi\sqrt{\frac{D}{2g}}$ (d) $2\pi\sqrt{\frac{2D}{g}}$

31. A toy car is made from a rectangle block of mass M and four disk wheels of mass m and radius r. The car is attached to a vertical wall by a massless horizontal spring with spring constant k and constrained to move perpendicular to the wall. The coefficient of static friction between the wheels of the car and the floor is μ. The maximum amplitude of oscillations of the car above which the wheels start slipping is [JEST 2017]

(a) $\frac{\mu g(M+2m)(M+4m)}{mk}$ (b) $\frac{\mu g(M^2-m^2)}{mk}$

(c) $\frac{\mu g(M+m)^2}{2mk}$ (d) $\frac{\mu g(M+4m)(M+6m)}{2mk}$

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32. A ball of mass 0.1 kg and density 2000 kg /m^3 is suspended by a massless string of length 0.5m under water having density 1000 kg /m^3. The ball experience a drag force, $\vec{F}_d = -0.2(\vec{v}_b - \vec{v}_w)$, where $\vec{v}_b$ and $\vec{v}_w$ are the velocities of the ball and water respectively. What will be the frequency of small oscillations for the motion of pendulum, if the water is at rest? [JEST 2017]

33. Three sinusoidal waves have the same frequency with amplitude $A, A/2$ and $A/3$ while their phase angles are $0, \pi/2$ and π respectively. The amplitude of the resultant wave is [JEST 2013]

(a) $11A/6$ (b) $2A/3$ (c) $5A/6$ (d) $7A/6$

34. A damped harmonic oscillator obeys the equation of motion $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$, with $\omega_0 = 2\pi rad/s$ and $\gamma = 5\pi s^{-1}$. The oscillator is started from rest with a displacement of 3 cm at $t = 0$. How many times is its speed equal to zero between $t = 0.25$ and $t = 10.25\ s$? [IISc. 2012]

(a) 0 (b) 2
(c) 10 (d) 20

35. The motion of a particle moving in a line is described by the equation $x(t) = 6 + 4\left(\cos\omega t + \frac{1}{16}(\cos 2\omega t - 1)\right)$. Given that n is an integer, the maximum displacement occurs at [IISc. 2010]

(a) $t = 2n\pi/\omega$ (b) $t = (2n+1)\pi/\omega$
(c) $t = \left(n + \frac{1}{2}\right)\pi/\omega$ (d) $t = \left(n + \frac{3}{4}\right)\pi/\omega$

36. A tunnel is dug from the surface of the earth through the center and opens at the other end. A ball is dropped from one end of the tunnel. The acceleration due to gravity on the earth's surface is g and the radius of the earth is R. Assuming that the earth has a constant density, the time taken by the ball to reach the center of the earth is [DU 2017]

(a) $\pi\sqrt{\frac{R}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}}$
(c) $\frac{\pi}{2}\sqrt{\frac{R}{g}}$ (d) $\frac{\pi}{4}\sqrt{\frac{R}{g}}$

37. Two springs of force constant K_1 and K_2 are arranged in a parallel arrangement and a mass m is suspended from it. The arrangement is equivalent to a single spring of constant K given by [DU 2015]

(a) $K_1 - K_2$ (b) $K_1 + K_2$

(c) $K_1K_2/(K_1+K_2)$ (d) K_1/K_2

38. A vertical spring, loaded with a mass, executes simple harmonic motion with a period of $4s$. The difference in the kinetic energy and potential energy of this system changes with a period of: [DU 2014]
(a) $4s$ (b) $5s$
(c) $8s$ (d) $16s$

39. The motion of a simple pendulum executing oscillations with large amplitude is : [DU 2014]
(a) Periodic and simple harmonic (b) Periodic but not simple harmonic
(c)Aperiodic (d) Rectilinear

40. A simple harmonic oscillator of mass m and frequency ω_0 is initially described by the equation: $\ddot{x}+\omega_0^2x=0$. A forcing force $mf_0e^{i\omega t}$ is now applied to drive the oscillator. The particular solution for x can in general be written as [HCU 2018]
(a) $\frac{f_0e^{i\omega t}}{(\omega_0^2+\omega^2)}$ (b) $\frac{f_0e^{i\omega t}}{(\omega_0^2+i\omega^2)}$
(c) $\frac{f_0e^{i\omega t}}{(\omega_0+i\omega)^2}$ (d) $\frac{f_0e^{i\omega t}}{(\omega_0^2-\omega^2)}$

41. If a point object moves in a plane according to $x=a\sin\omega t$ and $y=a\sin 2\omega t$, the trajectory equation of the object is given by [HCU 2018]
(a) $y(x)=\frac{2x(x^2-a^2)}{a^2}$ (b) $y(x)=x\sqrt{1-\frac{x^2}{a^2}}$
(c) $y(x)=\sqrt{1-\frac{x^2}{a^2}}$ (d) $y(x)=2x\sqrt{1-\frac{x^2}{a^2}}$

42. Consider a sonometer wire made by soldering two wires of radii r and $2r$ and kept under tension T Stationary vibrations are setup in the wire such that the soldered joint is midway between the two bridges and is a node. The ratio of the number of loops formed in the wires is [HCU 2018]
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

43. Consider a particle of mass m suspended between two identical springs. The mass m is displaced towards right by distance x (much smaller than l) as shown in figure and then left to oscillate. In leading order, the acceleration of the mass is proportional to [HCU 2018]

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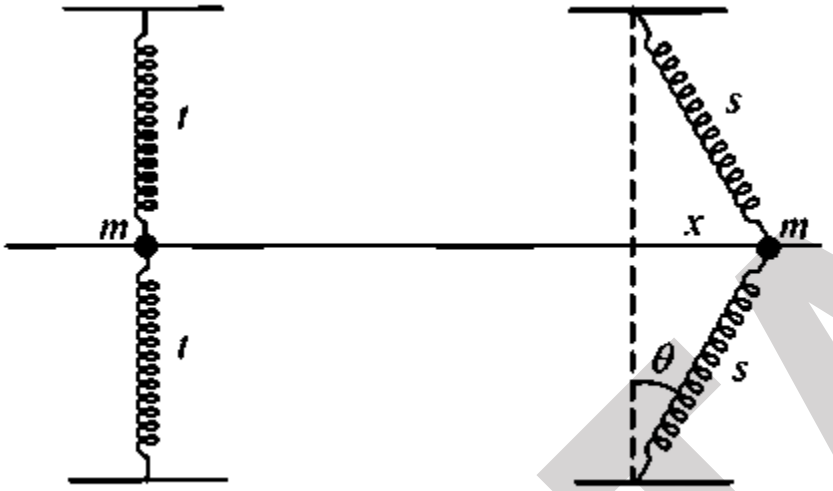


(a) Equilibrium position (b) Extended position

(a) $-x$ (b) $-x^2$
(c) $-x^3$ (d) $-e^{-x}$

44. A simple harmonic oscillator has velocities v_1 and v_2 at positions x_1 and x_2 (from the equilibrium point) respectively. The frequency of the oscillator is given by [HCU 2018]

(a) $\frac{(v_1^2+v_2^2)}{(x_1^2+x_2^2)}$ (b) $\frac{(v_1^2+v_2^2)}{(x_1^2-x_2^2)}$
(c) $\sqrt{\frac{(v_1^2-v_2^2)}{(x_2^2-x_1^2)}}$ (d) $\sqrt{\frac{(v_1^2-v_2^2)}{(x_1^2-x_2^2)}}$

45. One end of a spring is fixed to a rigid support and a body of mass m is attached to the other end such that the spring hangs vertically downwards. The unscratched length of the spring is l and it has a spring constant k. The extension of the spring (z) when the body is in equilibrium is [HCU 2016]

(a) $\frac{mg}{2k}$ (b) $-\frac{mg}{2k}$
(c) $\frac{mg}{k}$ (d) $-\frac{mg}{k}$

46. A block of mass is attached to a pair of springs connected in series. Both the springs have same spring constant k. The period of oscillations of the block is T_1. The period of oscillations of the same block attached to the same springs, connected in parallel is T_2. The ratio $T_1 : T_2$ is [HCU 2016]

(a) 2: 1 (b) 1: 2
(c) 4: 1 (d) 1: 4

47. Two sound sources oscillate in phase with a frequency of 100 Hz. At a point $5m$ from one source and $5.85m$ from the other the amplitude of the sound from each source separately is A. What is the phase difference of the two waves at that point (Assume that the speed of sound is $343m/s$.) [HCU 2015]

(a) 99^o (b) 45^o
(c) 60^o (d) 89^o

48. Two waves are represented by $E_1 = E_{10}\cos\omega t$ and $E_2 = E_{20}\sin(\omega t + \pi/2)$, where ω is the angular frequency and E_{10} and E_{20} are the amplitude of the waves. The superposition of these two waves leads to a resultant wave with an amplitude [HCU 2014]

(a) $\sqrt{(E_{10}^2 + E_{20}^2)}$ (b) $E_{10} + E_{20}$
(c) $\sqrt{(E_{10}^2 - E_{20}^2)}$ (d) $E_{10} - E_{20}$

49. The displacement of a body executing simple harmonic motion is given by

$$y = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$$

its initial displacement, assuming all quantities are in SI unit, is [HCU 2014]

(a) $\frac{5}{\sqrt{2}}m$ (b) $5m$
(c) $0m$ (d) $\frac{-5}{\sqrt{2}}m$

50. The maximum energy of a simple pendulum of length 1 m is E. If the length of the pendulum is increased to 1.5 m then its maximum energy, assuming amplitude has not changed, is [HCU 2014]

(a) $\frac{2}{3}E$ (b) $\frac{3}{2}E$
(c) $\sqrt{\frac{3}{2}}E$ (d) $\sqrt{\frac{2}{3}}E$

51. Two springs of springs constants k_1 and k_2 are attached to a block of mass m on its either side so as to be collinear. The free ends of both the springs are fixed to rigid supports. If the block is made to oscillate parallel to the springs then the frequency of oscillations is [HCU 2014]

(a) $\frac{1}{2\pi}\left(\frac{k_1+k_2}{m}\right)^{1/2}$ (b) $\left(\frac{k_1+k_2}{m}\right)^{1/2}$
(c) $\frac{1}{2\pi}\left(\frac{k_1-k_2}{m}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k_1+k_2}{2m}\right)^{1/2}$

QUANTA

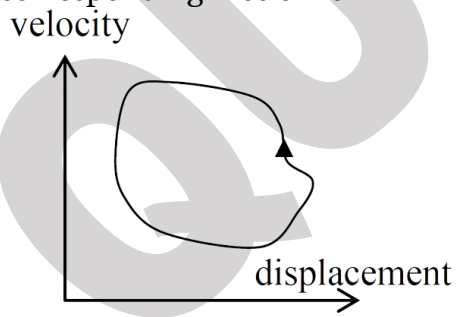
52. A simple pendulum executes oscillations of period 10s on the surface of Earth. This pendulum is taken to another planet having same mass but one fourth of Earth's size. The period of oscillations of the pendulum would now be [HCU 2014]
(a) 40 s (b) 10 s
(c) 5 s (d) 2.5 s

53. A body executes simple harmonic motion about the origin with amplitude $A = 13$ cm and period $T = 12$ sec. At time $t = 0$ it is at $x = 13$ cm. The shortest time of passage from $x = 6.5$ cm to $x = -6.5$ cm is [HCU 2013]
(a) 4 sec (b) 3 sec
(c) 2 sec (d) 1 sec

54. The superposition of two oscillations given by $y = A[\sin(\omega_1 t) + \sin(\omega_2 t)]$ is periodic if only the frequencies satisfy the relation [HCU 2013]
(a) ω_1/ω_2 is a real number (b) ω_1/ω_2 is an integer
(c) ω_1/ω_2 is rational (d) ω_1/ω_2 is irrational

55. For small vibrations of a compound pendulum of mass m. moment of inertia 1 and distance a of the center of mass from the origin O, will have time period [HCU 2013]
(a) $2\pi\sqrt{mga/I}$ (b) $\frac{1}{2\pi}\sqrt{I/mga}$
(c) $2\pi\sqrt{\frac{Iag}{m}}$ (d) $2\pi\sqrt{\frac{Ig}{am}}$

56. For certain motion the plot of velocity versus displacement is shown in figure. The corresponding motion is



(a) aperiodic (b) simple harmonic
(c) periodic (d) with constant acceleration

57. The fraction of kinetic energy in the total energy of a simple harmonic oscillator, when its displacement is half of its amplitude, is given by [HCU 2012]
(a) 0 (b) 0.5

(c)1 (d) 0.75

58. A particle of mass m, carrying a charge $+q$ hangs at rest from a spring of stiffness k. If a charge $-q$ is brought and held directly below $+q$ at a distance of d, then the change in length of the spring, Δz, in its new equilibrium state is proportional to [HCU 2012]

(a) d (b) $\frac{q^2}{d}\sqrt{\frac{k}{m}}$

(c) $\frac{q^2}{kd}$ (d) $\sqrt{\frac{kq^2}{md}}$

59. If a body of uniform cross-sectional area A and mass density ρ float in a liquid of density $\rho_0(\rho < \rho_0)$ and at equilibrium displaces a volume V of the liquid, then the time period of small oscillations about the equilibrium position is [HCU 2012]

(a) $T = \frac{1}{2\pi}\sqrt{\frac{V}{g\rho}}$ (b) $T = 2\pi(\rho - \rho_0)g\sqrt{\frac{V}{A}}$

(c) $T = \sqrt{\frac{gA}{V}}\frac{1}{(\rho-\rho_0)}$ (d) $T = 2\pi\sqrt{\frac{V}{gA}}$

60. A block of mass m rest on a frictionless horizontal table and is connected to two fixed posts by springs having spring constants k and $2k$. If the block is displaced from its equilibrium position, the angular frequency of vibrations is given by [HCU 2012]

(a) $\sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{k}{m}}$

(c) $\sqrt{\frac{k}{3m}}$ (d) $\sqrt{\frac{3k}{2m}}$

61. If a pendulum bob is suspended by a string of length L inside a fluid (the bob is η times more dense than the fluid), period of small oscillations of the pendulum will be given by [HCU 2011]

(a) $2\pi\sqrt{\eta L/g}$ (b) $2\pi\sqrt{\frac{\eta L}{g(\eta-1)}}$

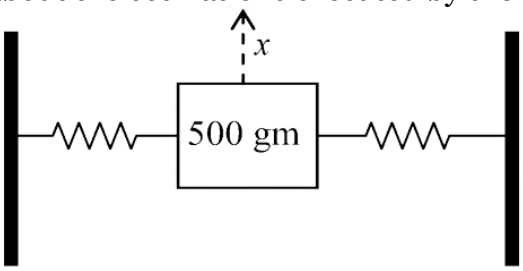
(c) $2\pi\sqrt{\frac{(\eta-1)L}{g\eta}}$ (d) $2\pi\sqrt{L/g\eta}$

62. A spring-mass system of mass M suspended from a spring constant k, and equilibrium stretching h, is equivalent to a simple pendulum of mass m and length [HCU 2011]

(a) $\frac{h}{4\pi^2}$ (b) h

(c) $\frac{h}{2}$ (d) $4\pi^2 h$

63. Consider a body of mass 500 gm connected to two springs as shown in the figure below. The springs are of equal length and spring constant. The body is displaced vertically upward by a small distance x and then left. Which of the following statements is true about the oscillations executed by the body? [HCU 2011]



(a) Motion is simple harmonic and periodic
(b) Motion is simple harmonic and not periodic
(c) Motion is not simple harmonic but periodic
(d) Motion is neither simple harmonic nor periodic

64. A simple pendulum, made of a point mass m attached to a massless string of length l, oscillates with a time period T_1. A second pendulum, in the shape of a thin uniform rod, has the same length l and the same total mass m and oscillates with the time period T_2. The thickness of the rod is negligible compared to l. The value of the ratio (T_2/T_1) is [JNU 2011]

(a) 0.44 (b) 0.67
(c) 0.82 (d) 1.00

65. A particle of mass m moves along the x-axis under the influence of the potential $V(x) = V_0(e^{-2ax} - 2e^{-ax})$. If the particle oscillates with small amplitude around the minimum of the potential, what is the period of oscillation? Take $m = 4$, $V_0 = 2$ and $\alpha = \frac{3}{4}$ (in appropriate units). [JNU 2011]

(a) 0.2 (b) 1.33
(c) 8.37 (d) 11.17

66. A uniform spring of spring constant k is cut into two pieces such that one piece is three times as long the other. The spring constant of the short piece is [JNU 2010]

(a) $k/4$ (b) k
(c) $4k/3$ (d) $4k$

67. The kinetic energy T and the potential energy V of a spring-mass system executing a simple harmonic motion vary with time. At an instant of time, at which the product TV

assumes its maximum value, let the displacement be a fraction f of the amplitude of oscillation. The absolute value of f is [JNU 2009]

(a) $1/\sqrt{3}$ (b) $1/\sqrt{2}$
(c) $1/3$ (d) ½

68. Consider the simple harmonic oscillator made up of a mass attached to a spring. What will be the effect on the motion if the mass were to stay completely immersed in a liquid? [JNU 2009]

(a) The motion is simple harmonic but with a frequency which is lower compared to when it is in air.
(b) The motion is always of the damped harmonic type.
(c) The motion is always overdamped
(d) The motion is either of the damped harmonic type, or it is overdamped, depending on the liquid.

69. An undamped, but forced harmonic oscillator satisfies the equation $\frac{d^2x}{dt^2} + \omega_0^2 x = f\cos\omega_0 t$ for the displacement $x(t)$ (f and ω_0 are constants). A solution for $x(t)$ is given by [ISM 2017]

(a) $\frac{f}{2\omega_0}\sin\omega_0 t$ (b) $\frac{f}{\omega_0} t\sin\omega_0 t$
(c) $\frac{f}{2\omega_0} t\sin\omega_0 t$ (d) $f\cos\omega_0 t$

70. Width of any resonance [BHU 2010]

(a) is directly proportional to the quality factor
(b) is inversely proportional to the quality factor
(c) does not depend on the quality factor
(d) does not depend on the natural frequency of the vibrating object

71. A sphere of radius r is kept on a concave mirror of radius of curvature R. The arrangement is kept on a horizontal table (the surface of concave mirror is frictionless and sliding not rolling). If the sphere is displaced from its equilibrium position and left, then it executes S.H.M. The period of oscillation will be (No rolling motion).

(a) $2\pi\sqrt{\left(\frac{(\mathrm{R-r})1.4}{\mathrm{g}}\right)}$ (b) $2\pi\sqrt{\left(\frac{R-r}{g}\right)}$
(c) $2\pi\sqrt{\left(\frac{\mathrm{rR}}{\mathrm{g}}\right)}$ (d) $2\pi\sqrt{\left(\frac{\mathrm{R}}{\mathrm{gr}}\right)}$

72. A simple pendulum is suspended from the roof of a train. The train is moving with an acceleration 49 cm/sec^2. By what angle to the vertical, its string will be inclined

(a) 20° (b) 30°
(c) Zero (d) 3° (approx.)

73. A pendulum of length 10 cm is suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination 60° with the horizontal. The time-period of oscillations of the pendulum will be
(a) $2\pi/7$sec (b) 2πsec
(c) πsec (d) $\pi/7$sec

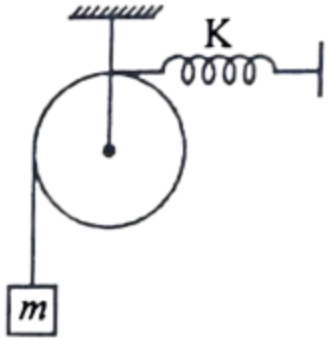
74. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface where R is the radius of the earth. The value of T_2/T_1 is
(a) 1 (b) $\sqrt{2}$
(c) 4 (d) 2

75. A particle of mass m is moving in a potential, energy $V(x) = \frac{1}{2}m\omega_0^2x^2 + \frac{a}{2mx^2}$ where, ω_0 and a are positive constants. The angular frequency of small oscillations for simple harmonic motion of the particle about a stable equilibrium of the potential $V(x)$ is
(a) $\sqrt{2}\omega_0$ (b) $2\omega_0$
(c) $4\omega_0$ (d) $4\sqrt{2}\omega_0$

76. The potential energy of a particle of mass 0.1 kg, moving along x-axis, is given by $U(x) = 5x(x-4)J$. where x is in meters. It can be concluded that
(a) the particle is acted upon by a constant force
(b) the speed of the particle is maximum at $x = 1$ m
(c) the particle does not executes SHM
(d) the period of oscillation is $\pi/5$sec

77. A particle of mass 10gm is placed in a potential field given by $V = (50x^2 + 100)$erg/gm. The angular frequency of oscillation is
(a) 10 (b) 1
(c) 5 (d) 4

78. A mass m is attached to a spring constant K via a friction-less pully of radius r and mass M as shown in figure. Frequency of the oscillation is



(a) $\omega = \sqrt{\frac{2K}{m}}$ (b) $\omega = \sqrt{\frac{K}{2m}}$

(c) $\omega = \sqrt{\frac{K}{m+M/2}}$ (d) $\omega = \sqrt{\frac{K}{2(m+M)}}$

79. A simple pendulum of equivalent length ℓ is kept immersed in a liquid of density η times less than the density of the spherical bob of the pendulum. Now the whole system is moved upward with acceleration g. The period of small oscillation of the pendulum is

(a) $T = 2\pi\sqrt{\frac{\ell}{(2\eta-2)g}}$ (b) $T = 2\pi\sqrt{\frac{\ell}{g}}$

(c) $T = 2\pi\sqrt{\frac{\ell}{2g}}$ (d) $T = 2\pi\sqrt{\frac{\eta L}{(2\eta-2)g}}$

80. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, lock A executes small oscillations the time period of which is given by

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{M\eta/L}$

(c) $2\pi\sqrt{ML/\eta A}$ (d) $2\pi\sqrt{M/\eta L}$

QUANTA

Answer Key:

1	1.09to1.11	21	A	41	D	61	B
2	D	22	D	42	C	62	B
3	A	23	D	43	B	63	C
4	A,C	24	B	44	D	64	C
5	A,D	25	A	45	D	65	C
6	A,B	26	C	46	A	66	D
7	B,C,D	27	C	47	D	67	B
8	B	28	A	48	B	68	D
9	C	29	D	49	A	69	C
10	B	30	C	50	A	70	B
11	0.25	31	D	51	A	71	B
12	A	32	3	52	D	72	D
13	D	33	C	53	C	73	A
14	D	34	A	54	*	74	D
15	C	35	C	55	B	75	B
16	B	36	C	56	C	76	D
17	B	37	B	57	D	77	A
18	A	38	B	58	*	78	C
19	A	39	B	59	D	79	D
20	C	40	D	60	A	80	d

QUANTA